CONDENSED LESSON 8.1

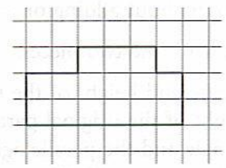
Areas of Rectangles and Parallelograms

In this lesson, you

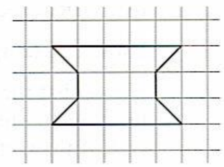
- Review the formula for the area of a rectangle
- Use the area formula for rectangles to find areas of other shapes
- Discover the formula for the area of a parallelogram

The **area** of a plane figure is the number of square units that can be arranged to fill the figure completely.

You probably already know several area formulas. The investigations in this chapter will help you understand and remember the formulas.



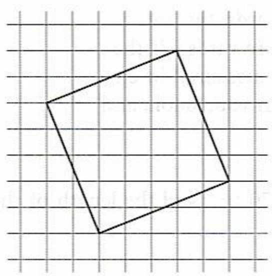
Area = 15 square units



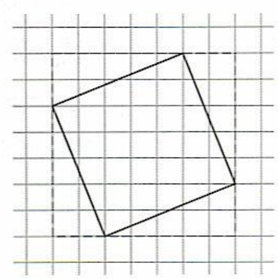
Area = 11 square units

Pages 410 and 411 of your book discuss the formula for the area of a rectangle. Read this text carefully. Make sure that you understand the meaning of **base** and **height** and that the area formula makes sense to you, then complete the Rectangle Area Conjecture in your book. Example A in your book shows how the area formula for rectangles can help you find areas of other shapes. Here is another example.

EXAMPLE A Find the area of this square.



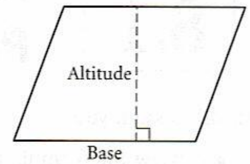
Solution Surround the given "slanted" square with a 7-by-7 square with horizontal and vertical sides. Then, subtract the area of the four right triangles formed from the area of the surrounding square.



Each of the four triangles is half of a 2-by-5 rectangle, so each has area $\frac{1}{2} \cdot 2 \cdot 5$, or 5 square units. Therefore, the area of the original square is $(7 \cdot 7) - (4 \cdot 5) = 29$ square units.

(continued)

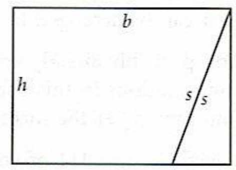
Just as with a rectangle, any side of a parallelogram can be called the *base.* However, the height of a parallelogram is not necessarily a side. Rather, the height is the length of an *altitude* to the base. An **altitude** is any segment from the side opposite the base, perpendicular to the line containing the base.



Investigation: Area Formula for Parallelograms

Follow Steps 1 and 2 of the investigation in your book. In Step 2, each new shape you form has the same area as the original parallelogram because you have simply rearranged pieces, without adding or removing any cardboard.

Form a rectangle with the two pieces.



Notice that the base and height of the rectangle are the same as the base and height of the original parallelogram. Because the area of the rectangle and the parallelogram are the same, the area of the parallelogram is bh. This can be summarized as a conjecture.

Parallelogram Area Conjecture The area of a parallelogram is given by the formula $A = bh$, where A is the area, b is the length of the base, and h is the height of the parallelogram. **C-76**

If the dimensions of a figure are measured in inches, feet, or yards, the area is measured in in.^2 (square inches), ft^2 (square feet), or yd^2 (square yards). If the dimensions are measured in centimeters or meters, the area is measured in cm^2 (square centimeters) or m^2 (square meters). Read Example B in your book, and then read the example below.

EXAMPLE B | A parallelogram has height 5.6 ft and area 70 ft^2. Find the length of the base.

▶ ***Solution***

$A = bh$	Write the formula.
$70 = b(5.6)$	Substitute the known values.
$\frac{70}{5.6} = b$	Solve for the base length.
$12.5 = b$	Divide.

The height measures 12.5 ft.

CONDENSED LESSON 8.2

Areas of Triangles, Trapezoids, and Kites

In this lesson, you

- Discover area formulas for triangles, trapezoids, and kites

You can use the area formulas you already know to derive new area formulas. In the first investigation, you'll focus on triangles.

Investigation 1: Area Formula for Triangles

Follow Step 1 in your book to create and label a pair of congruent triangles.

You know the area formula for rectangles and parallelograms. Arrange the two congruent triangles to form one of these figures. Write an expression for the area of the entire figure. Then, write an expression for the area of one of the triangles.

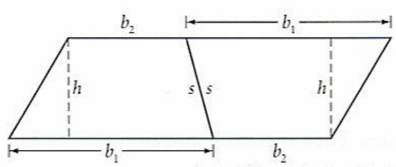
Summarize your findings by completing the conjecture below.

Triangle Area Conjecture The area of a triangle is given by the formula ________________, where A is the area, b is the length of the base, and h is the height of the triangle. **C-77**

Next, you'll consider the area of a trapezoid.

Investigation 2: Area Formula for Trapezoids

Follow Steps 1 and 2 in your book to make and label two congruent trapezoids. You can arrange the trapezoids to form a parallelogram.



What is the base length of the parallelogram? What is the height? Use your answers to the questions to write an expression for the area of the parallelogram. Then, use the expression for the area of the parallelogram to write an expression for the area of one trapezoid.

Summarize your findings by completing this conjecture.

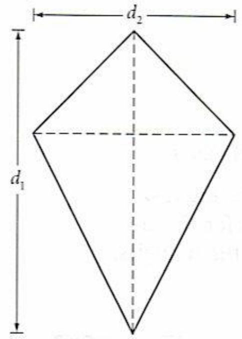
Trapezoid Area Conjecture The area of a trapezoid is given by the formula ________________, where A is the area, b_1 and b_2 are the lengths of the two bases, and h is the height of the trapezoid. **C-78**

(continued)

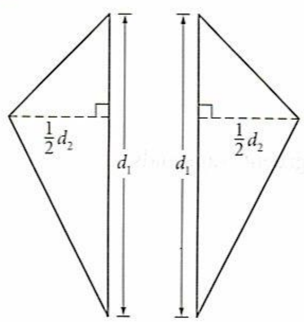
Finally, you will consider the area of a kite.

Investigation 3: Area Formula for Kites

Draw a kite. Draw its diagonals. Let d_1 be the length of the diagonal connecting the vertex angles, and let d_2 be the length of the other diagonal.



Recall that the diagonal connecting the vertex angles of a kite divides it into two congruent triangles. Consider the side labeled d_1 to be the base of one of the triangles. Then, because the diagonal connecting the vertex angles of a kite is the perpendicular bisector of the other diagonal, the height of the triangle is $\frac{1}{2}d_2$.



Write an expression for the area of one of the triangles. Then use the expression for the area of the triangle to write an expression for the area of the kite.

Summarize your findings by completing this conjecture.

Kite Area Conjecture The area of a kite is given by the formula ________________, where A is the area and d_1 and d_2 are the lengths of the diagonals. **C-79**

CONDENSED LESSON

8.3 Area Problems

In this lesson, you

- Use a variety of strategies to approximate the areas of irregularly shaped figures

You have discovered formulas for areas of rectangles, parallelograms, triangles, trapezoids, and kites. In this lesson, you will use these formulas, along with other methods, to find the approximate areas of irregularly shaped figures.

Investigation: Solving Problems with Area Formulas

On the next page, you'll find eight geometric figures. For each figure, find a way to calculate the approximate area. Then, record the area and write a sentence or two explaining how you found it.

Below are some hints for how you might find the area of each figure. Read these hints only if you get stuck. There are lots of ways to find each area. The methods you use may be very different from those described here.

Figure A Divide the figure into rectangles.

Figure B This figure is a kite. Use what you learned in Lesson 8.2 to find the area.

Figure C Divide the figure into triangles. Or surround the figure with a rectangle.

Figure D Divide the figure into triangles.

Figure E This figure is a trapezoid. Use what you learned in Lesson 8.2 to find the area.

Figure F Find the area of the two squares. Cut out the other two pieces and rearrange them to form a recognizable shape.

Figure G Divide this dodecagon into 12 identical, isosceles triangles with vertex angles at the "center" of the polygon.

Figure H Trace the figure onto a sheet of graph paper. Estimate the number of squares that fit inside the figure. Or draw as large a rectangle as will fit in the shape. Cut off the remaining pieces and arrange them to form recognizable shapes.

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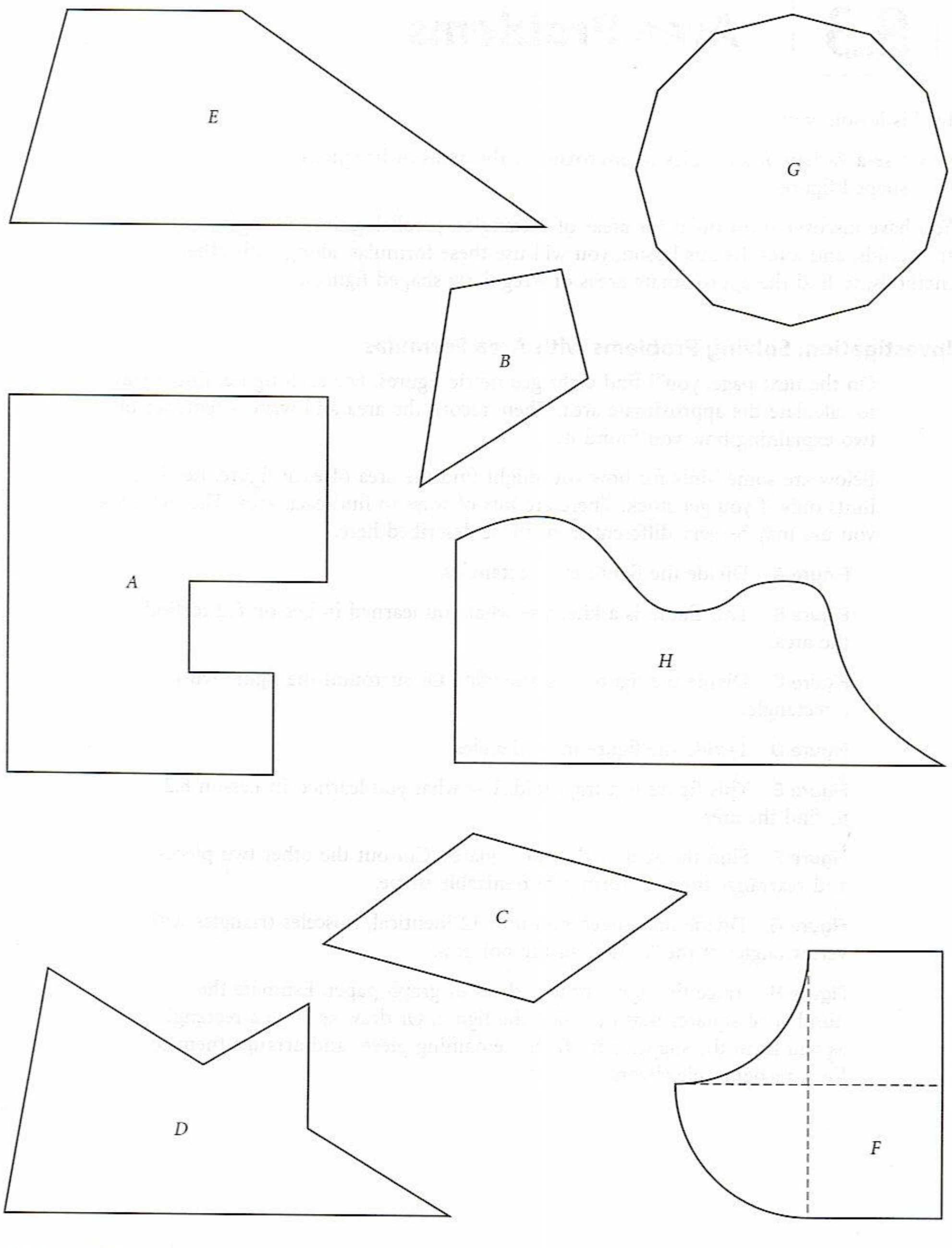
E
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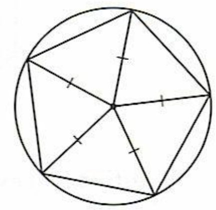
CONDENSED LESSON

8.4 Areas of Regular Polygons

In this lesson, you

- Discover the area formula for regular polygons



You can divide a regular polygon into congruent isosceles triangles by drawing segments from the center of the polygon to each vertex. The center of the polygon is actually the center of the circumscribed circle.

In the investigation, you will divide regular polygons into triangles. Then you will write a formula for the area of any regular polygon.

Investigation: Area Formula for Regular Polygons

The **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon. Follow the steps in your book to find the formula for the area of a regular n-sided polygon with sides of length s and apothem a. Your findings can be summarized in this conjecture.

Regular Polygon Area Conjecture The area of a regular polygon is given by the formula $A = \frac{1}{2}asn$, where A is the area, a is the apothem, s is the length of each side, and n is the number of sides. The length of each side times the number of sides is the perimeter, P, so $sn = P$. Thus you can also write the formula for area as $A = \frac{1}{2}aP$. **C-80**

The examples below show you how to apply your new formulas.

EXAMPLE A A regular nonagon has area 302.4 cm^2 and apothem 9.6 cm. Find the length of each side.

▶ ***Solution*** Because you are trying to find the side length, s, it is probably easier to use the formula $A = \frac{1}{2}asn$. You could also use $A = \frac{1}{2}aP$, solve for P, and then divide the result by 9 (the number of sides).

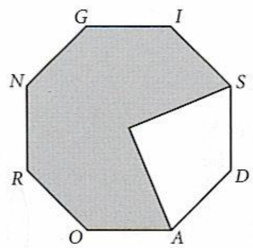
$A = \frac{1}{2}asn$	Write the formula.
$302.4 = \frac{1}{2}(9.6)(s)9$	Substitute the known values.
$302.4 = 43.2s$	Multiply.
$\frac{302.4}{43.2} = s$	Solve for s.
$7 = s$	Divide.

Each side has length 7 cm.

The following example is Exercise 12 in your book.

(continued)

EXAMPLE B Find the shaded area of the regular octagon *ROADSIGN*. The apothem measures about 20 cm. Segment *GI* measures about 16.6 cm.



Solution First, find the area of the entire octagon.

$A = \frac{1}{2}asn$ Write the formula.

$A \approx \frac{1}{2}(20)(16.6)(8)$ Substitute the known values.

$A \approx 1328$ Multiply.

The area of the octagon is about 1328 cm^2. The shaded portion makes up $\frac{6}{8}$ of the octagon. (If you divide the octagon into eight isosceles triangles, six will be shaded.) So, the shaded area is about $\frac{6}{8}(1328\ cm^2)$, or 996 cm^2.

CONDENSED LESSON

8.5 Areas of Circles

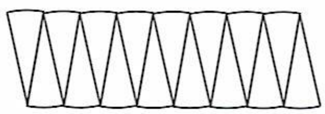
In this lesson, you

- Discover the area formula for circles

A rectangle has straight sides, while a circle is entirely curved. So, it may surprise you that you can use the area formula for rectangles to help you find the area formula for circles. In the next investigation, you'll see how.

Investigation: Area Formula for Circles

Follow Steps 1–3 in your book to create a figure like the one below.



The figure resembles a parallelogram. If you cut the circle into more wedges, you could arrange these thinner wedges to form a more rectangular parallelogram. You would not lose or gain any area in this change, so the area of this new "rectangle" would be the same as the area of the original circle. If you could cut infinitely many wedges, you'd actually have a rectangle with smooth sides.

The two longer sides of the rectangle would be made up of the circumference of the circle. Consider one of these sides to be the base.

Write an expression for the length of the base of the rectangle. What is the height of the rectangle? What is the area of the rectangle?

Remember, the area of the rectangle is the same as the area of the original circle. Use this idea and your findings to complete this conjecture.

Circle Area Conjecture The area of a circle is given by the formula $A =$ ________________, where A is the area and r is the radius of the circle. **C-81**

Examples A and B in your book show you how to use your new conjecture. Read these examples carefully, and then read the examples on the next page.

(continued)

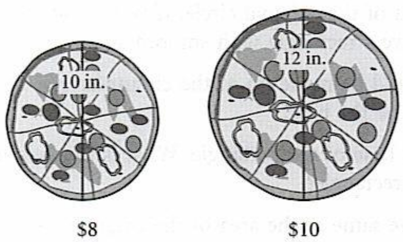
EXAMPLE A The circumference of a circle is 22π ft. What is the area of the circle?

▸ **Solution** Use the circumference formula to find the radius. Then, use the area formula to find the area.

$C = 2\pi r$	Write the formula for circumference.
$22\pi = 2\pi r$	Substitute the known values.
$11 = r$	Solve for r.
$A = \pi r^2$	Write the formula for area.
$A = \pi(11)^2$	Substitute the known values.
$A = 121\pi$	Simplify.

The area is 121π ft^2, or about 380.1 ft^2.

EXAMPLE B At Maria's Pizzeria, a pepperoni pizza with diameter 10 inches costs \$8, and a pepperoni pizza with diameter 12 inches costs \$10. Which size costs less per square inch?



▸ **Solution** Find the area of each pizza, and then find the price per square inch.

10-inch pizza

$A = \pi r^2$

$= \pi(5)^2$

$= 25\pi$

The area is 25π in.^{2}. To find the cost per square inch, divide the price by the area.

$\frac{8}{25\pi} \approx 0.10$

The 10-inch pizza costs about 10¢ per square inch.

12-inch pizza

$A = \pi r^2$

$= \pi(6)^2$

$= 36\pi$

The area is 36π in.^{2}. To find the cost per square inch, divide the price by the area.

$\frac{10}{36\pi} \approx 0.09$

The 12-inch pizza costs about 9¢ per square inch.

The 12-inch pizza costs less per square inch.

CONDENSED LESSON

8.6 Any Way You Slice It

In this lesson, you

- Learn how to find the area of a sector, a segment, and an annulus of a circle

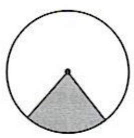
In Lesson 8.5, you discovered the formula for calculating the area of a circle. In this lesson, you'll learn how to find the areas of three types of sections of a circle.

A **sector of a circle** is the region between two radii of a circle and the included arc.

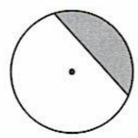
A **segment of a circle** is the region between a chord of a circle and the included arc.

An **annulus** is the region between two concentric circles.

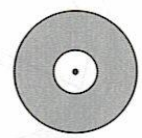
Examples of the three types of sections are pictured below.



Sector of a circle

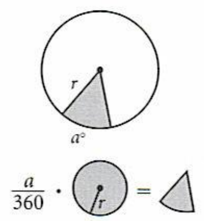


Segment of a circle

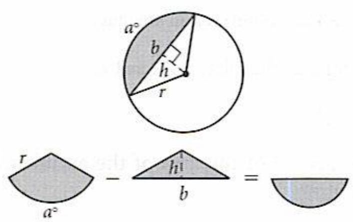


Annulus

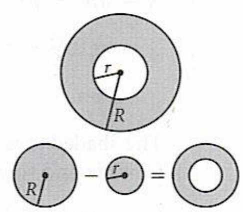
The "picture equations" below show you how to calculate the area of each type of section.



$\left(\frac{a}{360}\right) \cdot \pi r^2 = A_{\text{sector}}$



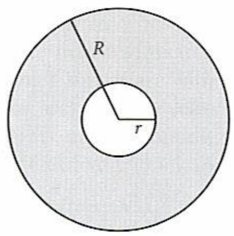
$\left(\frac{a}{360}\right) \cdot \pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$



$\pi R^2 - \pi r^2 = A_{\text{annulus}}$

Read the examples in your book carefully. Then read the examples below.

EXAMPLE A $R = 9$ cm and $r = 3$ cm. Find the area of the annulus.



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▶ ***Solution***

$A = \pi R^2 - \pi r^2$ — The area formula for an annulus.

$= \pi(9)^2 - \pi(3)^2$ — Substitute the values for R and r.

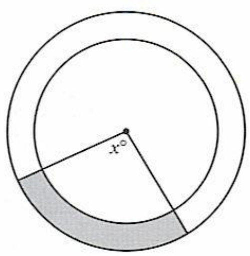
$= 81\pi - 9\pi$ — Evaluate the exponents.

$= 72\pi$ — Subtract.

The area of the annulus is 72π cm^2.

The example below is Exercise 12 in your book.

EXAMPLE B The shaded area is 10π cm^2. The radius of the large circle is 10 cm, and the radius of the small circle is 8 cm. Find x, the measure of the central angle.



▶ ***Solution*** First, find the area of the whole annulus.

$A = \pi R^2 - \pi r^2$ — The area formula for an annulus.

$= \pi(10)^2 - \pi(8)^2$ — Substitute the values for R and r.

$= 36\pi$ — Simplify.

The shaded area, 10π cm^2, is $\frac{x}{360}$ of the area of the annulus. Use this information to write and solve an equation.

$$10\pi = \frac{x}{360} \cdot 36\pi$$

$$360 \cdot \frac{10\pi}{36\pi} = x$$

$$100 = x$$

The measure of the central angle is 100°.

CONDENSED LESSON

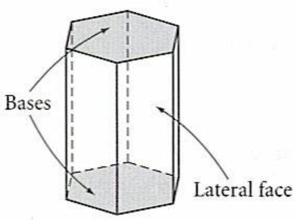
8.7 Surface Area

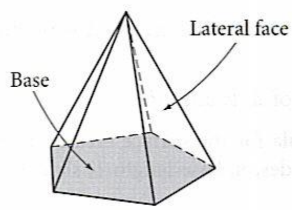
In this lesson, you

- Learn how to find the surface areas of prisms, pyramids, cylinders, and cones

You can use what you know about finding the areas of plane figures to find the surface areas of prisms, pyramids, cylinders, and cones. The **surface area** of each of these solids is the sum of the areas of all the faces or surfaces that enclose the solid. The faces include the solid's **bases** and its remaining **lateral faces.**

In a prism, the bases are two congruent polygons and the lateral faces are rectangles or parallelograms. In a pyramid, the base can be any polygon. The lateral faces are triangles.

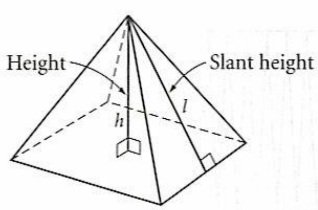




Read the "Steps for Finding Surface Area" on page 446 of your book. Example A shows how to find the surface area of a rectangular prism. Read the example carefully.

Then, read Example B, which shows how to find the surface area of a cylinder. Notice that, to find the area of the cylinder's lateral surface, you need to imagine cutting the surface and laying it flat to get a rectangle. Because the rectangle wraps exactly around the circular base, the length of the rectangle's base is the circumference of the circular base.

The surface area of a pyramid is the area of the base, plus the areas of the triangular faces. The height of each triangular face is called the **slant height.** To avoid confusing slant height with the height of the pyramid, use l rather than h for slant height.



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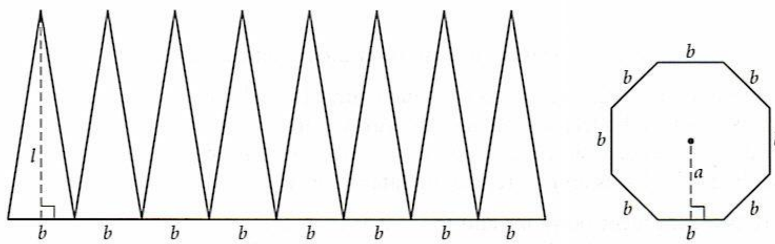
Investigation 1: Surface Area of a Regular Pyramid

The lateral faces of a regular pyramid are identical triangles, and the base is a regular polygon.



Each lateral surface is a triangle with base length b and height l. What is the area of each face?

If the base is an n-gon, then there are n lateral faces. What is the total lateral surface area of the pyramid?

What is the area of the base in terms of a, b, and n?

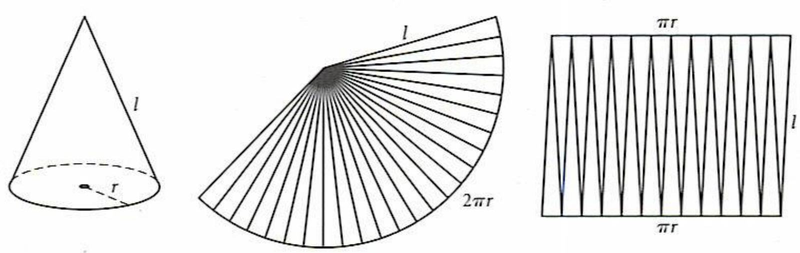
Use your expressions to write a formula for the surface area of a regular n-gon pyramid in terms of the number of sides, n, base length b, slant height l, and apothem a.

Using the fact that the perimeter of the base is nb, write another formula for the surface area of a regular n-gon pyramid in terms of slant height l, apothem a, and perimeter of the base, P.

In the next investigation, you will find the surface area of a cone with radius r and slant height l.

Investigation 2: Surface Area of a Cone

As the number of faces of a pyramid increases, it begins to look like a cone. You can think of the lateral surface as many thin triangles, or as a sector of a circle. You can rearrange the triangles to form a rectangle.



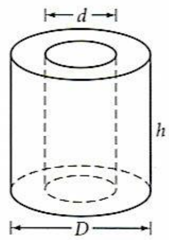
Use the diagrams to help you write a formula for the lateral surface area in terms of r and l.

Using the expression for the lateral surface area and an expression for the area of the base, write a formula for the surface area of the cone.

(continued)

Example C in your book shows you how to apply the formula for the surface area of a cone. Read this example carefully. The example below is Exercise 9 in your book.

EXAMPLE Find the surface area of this solid. $D = 8$, $d = 4$, $h = 9$.



Solution The surface area is the lateral surface area of the outside cylinder, plus the lateral surface area of the inside cylinder, plus the area of the two bases, which are annuluses.

$$\text{Lateral surface area of outside cylinder} = 2\pi\left(\frac{D}{2}\right)h = 2\pi\left(\frac{8}{2}\right)9 = 72\pi \text{ cm}^2$$

$$\text{Lateral surface area of inside cylinder} = 2\pi\left(\frac{d}{2}\right)h = 2\pi\left(\frac{4}{2}\right)9 = 36\pi \text{ cm}^2$$

$$\text{Area of one base} = \pi\left(\frac{D}{2}\right)^2 - \pi\left(\frac{d}{2}\right)^2$$

$$= \pi\left(\frac{8}{2}\right)^2 - \pi\left(\frac{4}{2}\right)^2 = 12\pi \text{ cm}^2$$

So,

$$\text{Total surface area} = 72\pi + 36\pi + 2(12\pi) = 132\pi \text{ cm}^2 \approx 414.7 \text{ cm}^2.$$